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Anisotropic Bianchi Type-III Bulk Viscous String Cosmological Model in Presence of Magnetic Field

Abstract : In this paper, we investigate a Bianchi Type-III string cosmological model in the presence of bulk viscosity and magnetic field within the framework of general relativity. The model is constructed to study the combined influence of dissipative effects and magnetic fields on the evolution of an anisotropic universe. The energy-momentum tensor is taken for a cloud of strings with bulk viscosity together with an electromagnetic field contribution. To obtain exact solutions of the Einstein field equations, Takabayasi's equation of state relating the energy density and string tension density is assumed. Further, the condition that the scalar expansion is proportional to the shear scalar is imposed, leading to a definite relationship among the metric potentials. Exact solutions of the field equations are obtained for the Bianchi Type-III space-time geometry.

The physical and geometrical properties of the model are analysed in detail through the behaviour of important cosmological parameters such as energy density, string tension density, particle density, bulk viscosity coefficient, expansion scalar, shear scalar, and volume scale factor. The analysis shows that the universe starts with an initial singularity and expands with time. The magnetic field significantly influences the expansion dynamics and anisotropic behaviour of the universe. Bulk viscosity introduces dissipative effects that modify the cosmic evolution and affect the decay of anisotropies. It is observed that the ratio of shear scalar to expansion scalar does not vanish at late times, indicating that the model remains anisotropic throughout its evolution. Special cases of the model corresponding to the absence of magnetic field and geometric string solutions are also discussed. The present study provides a useful framework for understanding the role of cosmic strings, magnetic fields, and viscous effects in the dynamics of the early universe.

Keywords : Bianchi Type-III space-time; Cosmic strings; Bulk viscosity; Magnetic field; Anisotropic universe; Einstein field equations; String cosmology; Shear scalar; Expansion scalar; Exact solutions; Early universe; General relativity.

1. Introduction : In this paper, we investigate a Bianchi Type-III cosmological model in the presence of bulk viscous strings and a magnetic field. The main objective is to study the combined effects of bulk viscosity and magnetic fields on the dynamics and evolution of an anisotropic universe. Since the Bianchi Type-III space-time permits directional dependence in cosmic expansion, it provides a suitable framework for examining anisotropic phases of the early universe.

Bulk viscosity is introduced to represent dissipative effects arising from internal friction within the cosmic fluid. Such dissipative processes are believed to have played an important role during the early stages of cosmic evolution. In addition, the inclusion of a magnetic field aligned along the direction of the strings enhances the physical relevance of the model because primordial magnetic fields are expected to have influenced the evolution of the universe. The interaction among string matter, viscous effects, and magnetic fields provides deeper insight into the behaviour of the early anisotropic universe.

In modern cosmology, string cosmological models have attracted considerable attention because of their possible role in explaining several phenomena associated with the early universe [**1-5**]. Cosmic strings are hypothetical one-dimensional topological defects that may have originated during symmetry-breaking phase transitions in the primordial universe. These strings possess energy and tension and therefore interact gravitationally with the surrounding space-time. Consequently, they are considered important candidates in the formation of large-scale structures through the generation of density perturbations [**6**].

The relativistic formulation for a cloud of strings was first introduced by Letelier [**7**], while Stachel [**8**] further developed the concept. Letelier studied string distributions in anisotropic cosmological models such as Bianchi Type-I and Kantowski–Sachs space-times. Later, several researchers extended these investigations to more general anisotropic geometries. Krori et al. [**9-10**] and Wang [**11-13**] examined string cosmological models in Bianchi Types I, VI, VIII, and IX space-times. Tikekar and Patel [**14**] obtained exact solutions for Bianchi Type-III models, whereas Chakraborty and Chakraborty [**15**] discussed exact solutions for spherically symmetric string cloud models. These studies demonstrate the importance of string cosmology in understanding non-trivial gravitational phenomena and anisotropic cosmic evolution.

Another important aspect of the early universe is the presence of dissipative processes. During the early epochs, especially near neutrino decoupling, cosmic matter is expected to have behaved like a viscous fluid. Bulk viscosity modifies the effective pressure of the cosmic medium and significantly affects the expansion dynamics of the universe. It may also contribute to entropy generation and smoothing of anisotropies. Motivated by these considerations, several authors have incorporated bulk viscosity into string cosmological models in different Bianchi space-times [**11-12,16-25**]. These investigations have provided a more realistic description of the anisotropic and dissipative behaviour of the early universe.

Magnetic fields also play an essential role in cosmology. Observational evidence indicates that magnetic fields exist on both galactic and intergalactic scales, suggesting their importance in cosmic evolution. Magnetic fields influence several astrophysical processes, including galaxy formation, cosmic ray propagation, and large-scale structure dynamics. Melvin [**26**] pointed out that matter in the early universe was highly ionized and strongly coupled with magnetic fields. As the universe expanded and cooled, recombination led to the formation of neutral matter; however, before recombination, magnetic fields would have significantly affected the behaviour of cosmic matter. This provides strong motivation for incorporating magnetic fields into cosmological models involving string clouds.

Several investigators have therefore studied magnetic fields in anisotropic string cosmologies. The coexistence of magnetic fields and cosmic strings has been analysed by many researchers [**27-29**]. Important contributions in this direction were made by Bahera [30], Bali and Dave [**31**], Bali [**32**], Kibble [**33**], and Takabayashi [**34**]. These works explored the influence of magnetic fields on string matter, anisotropic expansion, and cosmic dynamics. Later, Yadav et al. [**35**] and Zimdahl [**36**] extended such studies by including additional effects such as viscosity and dark energy in magnetic string cosmological models. Altogether, these investigations suggest that magnetic fields constitute an important ingredient in understanding the behaviour and evolution of the early universe.

In the present chapter, we construct and analyse a Bianchi Type-III cosmological model filled with a bulk viscous string cloud in the presence of a magnetic field. Since Bianchi Type-III space-time is anisotropic in nature, it allows us to examine the possibility that the early universe expanded differently along different spatial directions. Such anisotropic models are useful for understanding deviations from perfect isotropy during the primordial stages of cosmic evolution.

To obtain deterministic solutions of the Einstein field equations, we impose the following assumptions:

1. We consider Takabayasi's equation of state relating the energy density $\rho$and string tension density λin the form

$$\boldsymbol{\rho = D + L\lambda},$$

where $D$and $L$are constants.

2. We assume that the scalar expansion $\theta$is proportional to the shear scalar σ, i.e.,

$$\boldsymbol{\sigma \propto \theta}.$$

   This condition leads to a specific relation among the metric potentials and simplifies the field equations considerably.

Using these assumptions, exact solutions of the Einstein field equations are obtained for the Bianchi Type-III geometry. The physical and geometrical behaviour of the model is then analysed both in the presence and absence of magnetic fields. The magnetic field contributes anisotropic pressure and influences the rate of expansion in different spatial directions. On the other hand, the absence of magnetic fields allows us to examine the independent effects of bulk viscosity and string matter on the cosmic evolution.

Various physical parameters such as the energy density, string tension density, expansion scalar, shear scalar, and volume scale factor are discussed in detail. The geometrical properties of the model are also examined to determine whether the universe approaches isotropy at late times or continues to remain anisotropic. Hence, the present study provides a comprehensive description of an anisotropic universe influenced by bulk viscosity, cosmic strings, and magnetic fields, and contributes to the understanding of the physical conditions prevailing in the early universe.

2. The Field Equations :

We consider the Bianchi Type-III space-time metric in the form

$$\boldsymbol{ds^2 = -dt^2 + \alpha^2 dx^2 + \beta^2 e^{-2kx} dy^2 + \gamma^2 dz^2}, \qquad \textbf{(2.1)}$$

where α, β, and $\gamma$are functions of cosmic time $t$only, and $k$is a constant.

The energy-momentum tensor for a cloud of strings with bulk viscosity and magnetic field is taken as [9]

$$\boldsymbol{T_{ij} = \rho u_i u_j - \lambda X_i X_j - \zeta\theta(u_i u_j + g_{ij}) + E_{ij},} \qquad \textbf{(2.2)}$$

where $\rho$is the proper energy density, $\lambda$is the string tension density, ζdenotes the coefficient of bulk viscosity, and θrepresents the scalar expansion. The total energy density is given by

$$\boldsymbol{\rho = \rho_p + \lambda,}$$

where $\rho_p$is the particle energy density attached to the strings.

The four-velocity vector $u^i$and the direction vector X^isatisfy the conditions

$$\boldsymbol{u^i u_i = -1, X^i X_i = 1, u^i X_i = 0.} \qquad \textbf{(2.3)}$$

The term E_{ij}denotes the electromagnetic energy-momentum tensor and is defined by

$$\boldsymbol{E_{ij} = \frac{1}{4\pi}\left(g^{hk} F_{ih} F_{jk} - \frac{1}{4} g_{ij} F_{hk} F^{hk}\right),} \qquad \textbf{(2.4)}$$

where $F_{ij}$is the electromagnetic field tensor satisfying Maxwell's equations

$$\boldsymbol{F_{[ij;k]} = 0, (F^{ij}\sqrt{-g})_{;j} = 0.} \qquad \textbf{(2.5)}$$

The Einstein field equations are taken as

$$\boldsymbol{R_{ij} - \frac{1}{2} R g_{ij} = T_{ij},} \qquad \textbf{(2.6)}$$

using natural units $c = 1$and $8\pi G = 1$.

For a co-moving coordinate system, we choose

$$\boldsymbol{u^i = \delta^i_0, u_i = -\delta^0_i.}$$

Assuming that the magnetic field is directed along the z-axis, Maxwell's equations imply that the only non-vanishing component of the electromagnetic field tensor is

$$\boldsymbol{F_{12} = H = \text{constant}, F_{14} = 0,} \qquad \textbf{(2.7)}$$

where Hrepresents the magnetic field intensity.

Using the metric (2.1), the Einstein field equations reduce to the following system of equations [**27,28**]:

$$\boldsymbol{\frac{\ddot{\beta}}{\beta} + \frac{\ddot{\gamma}}{\gamma} + \frac{\dot{\beta}\dot{\gamma}}{\beta\gamma} = \zeta\theta - \frac{H^2}{8\pi\alpha^2\beta^2 e^{-2kx}},} \qquad \textbf{(2.8)}$$

$$\boldsymbol{\frac{\ddot{\alpha}}{\alpha} + \frac{\ddot{\gamma}}{\gamma} + \frac{\dot{\alpha}\dot{\gamma}}{\alpha\gamma} = \zeta\theta - \frac{H^2}{8\pi\alpha^2\beta^2 e^{-2kx}},} \qquad \textbf{(2.9)}$$

$$\boldsymbol{\frac{\ddot{\alpha}}{\alpha} + \frac{\ddot{\beta}}{\beta} + \frac{\dot{\alpha}\dot{\beta}}{\alpha\beta} - \frac{1}{\alpha^2} = \lambda + \zeta\theta + \frac{H^2}{8\pi\alpha^2\beta^2 e^{-2kx}},} \qquad \textbf{(2.10)}$$

$$\boldsymbol{\frac{\dot{\alpha}\dot{\beta}}{\alpha\beta} + \frac{\dot{\beta}\dot{\gamma}}{\beta\gamma} + \frac{\dot{\alpha}\dot{\gamma}}{\alpha\gamma} - \frac{1}{\alpha^2} = \rho + \frac{H^2}{8\pi\alpha^2\beta^2 e^{-2kx}},} \qquad \textbf{(2.11)}$$

$$\boldsymbol{\frac{\dot{\alpha}}{\alpha} - \frac{\dot{\beta}}{\beta} = 0.} \qquad \textbf{(2.12)}$$

Here, an overhead dot denotes differentiation with respect to cosmic time t.

For $k = -1$, the metric assumes the form [37]

$$\boldsymbol{ds^2 = -dt^2 + \alpha^2 dx^2 + \beta^2 e^{2x} dy^2 + \gamma^2 dz^2.} \qquad \textbf{(2.13)}$$

Accordingly, the field equations become

$$\frac{\ddot{\beta}}{\beta}+\frac{\ddot{\gamma}}{\gamma}+\frac{\dot{\beta}\dot{\gamma}}{\beta\gamma}=\zeta\theta-\frac{H^2}{8\pi\alpha^2\beta^2 e^{2x}}, \qquad (2.14)$$

$$\frac{\ddot{\alpha}}{\alpha}+\frac{\ddot{\gamma}}{\gamma}+\frac{\dot{\alpha}\dot{\gamma}}{\alpha\gamma}=\zeta\theta-\frac{H^2}{8\pi\alpha^2\beta^2 e^{2x}}, \qquad (2.15)$$

$$\frac{\ddot{\alpha}}{\alpha}+\frac{\ddot{\beta}}{\beta}+\frac{\dot{\alpha}\dot{\beta}}{\alpha\beta}-\frac{1}{\alpha^2}=\lambda+\zeta\theta+\frac{H^2}{8\pi\alpha^2\beta^2 e^{2x}}, \qquad (2.16)$$

$$\frac{\dot{\alpha}\dot{\beta}}{\alpha\beta}+\frac{\dot{\beta}\dot{\gamma}}{\beta\gamma}+\frac{\dot{\alpha}\dot{\gamma}}{\alpha\gamma}-\frac{1}{\alpha^2}=\rho+\frac{H^2}{8\pi\alpha^2\beta^2 e^{2x}}, \qquad (2.17)$$

$$\frac{\dot{\alpha}}{\alpha}-\frac{\dot{\beta}}{\beta}=0. \qquad (2.18)$$

Integrating equation (2.18), we obtain

$$\alpha=K\beta, \qquad (2.19)$$

where $K$is an integration constant.

To obtain a determinate solution of the field equations, we further assume Takabayasi's equation of state [34] in the form

$$\rho=D+L\lambda, \qquad (2.20)$$

where $D$and $L$are constants.

3. Discussion : From Equations (2.1) and (2.3), it follows that the physical requirements of positive energy density $(\rho\geq 0)$and positive particle density $\left(\rho_p\geq 0\right)$are satisfied under certain restrictions on the parameters involved in the model. These conditions hold when

$$\Psi\geq 0, \mu>0, \text{ and } L>\frac{(\mu+1)(\mu+3)}{\mu}$$

or

$$\Psi\geq 0, \mu>0, \text{ and } L<-\frac{(\mu+1)(2\mu+5)}{}$$

For the first range of values of L, the string tension density λremains positive, whereas for the second range it becomes negative. Hence, the sign and nature of the string tension are strongly governed by the parameter L.

The expressions obtained for ρ, λ, ρ_p, ζ, θ, and σ^2clearly show the contribution of the magnetic field through the parameter H. Therefore, the magnetic field directly affects both the geometrical evolution and the physical characteristics of the cosmological model. The additional magnetic term modifies the behavior of matter density, viscous effects, and the expansion dynamics of the universe.

For $\mu>0$, irrespective of whether

$$L>\frac{(\mu+1)(\mu+3)}{\mu}$$

or

$$L<-\frac{(\mu+1)(2\mu+5)}{}$$

we obtain $l+L>0$. Consequently, from Equation (2.5), the expansion scalar θbecomes infinitely large as $\tau\to 0$. This indicates the presence of an initial singularity corresponding to the origin of the universe. On the other hand, as $\tau\to\infty$, the expansion scalar gradually decreases and approaches either a finite value or zero, showing that the rate of cosmic expansion slows down with time.

The energy density ρalso exhibits a similar temporal behavior. It diverges near $\tau = 0$, implying that the universe initially existed in a highly dense state. As time progresses, the density decreases continuously and tends toward a finite value at late times. Such behavior supports the Big Bang picture of cosmological evolution, where the universe starts from an extremely hot and dense phase and expands gradually.

The model represents an anisotropic, expanding, and shearing universe without rotation. Bulk viscosity plays a significant role in determining the dynamics of the cosmic evolution. The presence of viscosity introduces dissipative effects that influence both the expansion rate and the decay of anisotropies in the early universe. These viscous effects are important in understanding thermodynamic processes during the early stages of cosmic evolution.

An important feature of the present model is that the ratio

$$\frac{\sigma}{\theta}$$

does not vanish as $\tau \to \infty$. This means that the anisotropy of the universe persists even at late times. Hence, the model does not evolve toward isotropy, unlike the standard Friedmann cosmological models. The continued existence of anisotropy demonstrates that the initial anisotropic conditions, together with the influence of bulk viscosity and magnetic field, remain significant throughout the cosmic evolution.

A special case arises when $L = 1$. Under this condition, the cosmological model reduces to a pure geometric string model. Furthermore, if the magnetic field is absent $(H = 0)$, the metric represented by Equation (2.12) simplifies to the Bianchi type-III string cosmological model with bulk viscosity only. In this case, the metric takes the form

$$\boldsymbol{ds^2 = -\left[\Psi\tau^{-2l} + (L-1)\{L(\mu^2+2\mu) - (\mu^2+4\mu+3)\}K^2\tau^{-2\mu}\right]^{-1} d\tau^2}$$
$$\boldsymbol{+K^2\tau^{2\mu+2}dx^2 + \tau^{2\mu+2}e^{2x}dy^2 + \tau^2 dz^2}$$

Thus, the present investigation generalizes earlier string cosmological models by incorporating both magnetic field effects and bulk viscous phenomena within the framework of Bianchi type-III space-time.

Conclusion : In this paper, a Bianchi type-III string cosmological model in the presence of bulk viscosity and magnetic field has been investigated within the framework of general relativity. To obtain exact solutions of the Einstein field equations, the physically motivated assumption that the scalar of shear is proportional to the scalar of expansion $(\sigma \propto \theta)$has been employed. An equation of state of the form

$$\boldsymbol{\rho = A + L\lambda}$$

has also been considered, which leads to a functional relation among the metric potentials given by

$$\boldsymbol{\beta = A + B\gamma^{\mu+1}}$$

Using these assumptions, explicit expressions for the metric functions and other physical parameters of the model have been obtained.

The analysis reveals that the universe described by this model is anisotropic, expanding, and shearing, but free from rotation. The behavior of the expansion scalar and energy density indicates the existence of an initial singularity, suggesting that the universe originated from a Big Bang type state. As cosmic time increases, the expansion rate decreases gradually, while the energy density also diminishes.

The study further demonstrates that the magnetic field significantly affects the dynamical behavior of the universe. The magnetic contribution appears explicitly in the expressions for the energy density, string tension density, particle density, bulk viscosity, expansion scalar, and shear scalar. Therefore, the magnetic field plays an important role in shaping the physical and geometrical evolution of the cosmological model.

Bulk viscosity also contributes substantially to the evolution of the universe by introducing dissipative mechanisms. These effects influence the thermodynamic behavior of the cosmic fluid and modify the rate of expansion. However, despite the presence of viscosity, the ratio σ/θdoes not vanish at large times, showing that the model does not approach isotropy during its evolution.

A particular case of the model has also been discussed. When $L = 1$, the solution reduces to a geometric string model, and in the absence of magnetic field $(H = 0)$, the model simplifies to the Bianchi type-III string cosmological model with bulk viscosity only.

Hence, the present work provides a comprehensive description of the combined influence of strings, bulk viscosity, and magnetic field on the evolution of an anisotropic universe. The obtained solutions may be useful for understanding the role of dissipative effects and magnetic fields in the dynamics of the early universe.

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